Construction of Variable Strength Covering Array for Combinatorial Testing Using a Greedy Approach to Genetic Algorithm

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Abstract

The limitation of time and budget usually prohibits exhaustive testing of interactions between components in a component based software system. Combinatorial testing is a software testing technique that can be used to detect faults in a component based software system caused by the interactions of components in an effective and efficient way. Most of the research in the field of combinatorial testing till now has focused on the construction of optimal covering array (CA) of fixed strength $t$ which covers all $t$-way interactions among components. The size of CA increases with the increase in strength of testing $t$, which further increases the cost of testing. However, not all components require higher strength interaction testing. Hence, in a system with $k$ components a technique is required to construct CA of fixed strength $t$ which covers all $t$-way interactions among $k$ components and all $t_i$-way (where $t_i > t$) interactions between a subset of $k$ components. This is achieved using the variable strength covering array (VSCA). In this paper we propose a greedy based genetic algorithm (GA) to generate optimal VSCA. Experiments are conducted on several benchmark configurations to evaluate the effectiveness of the proposed approach.

Keywords: combinatorial testing, variable strength covering array, genetic algorithm, greedy approach

1. Introduction

The increasing dependence on software systems in every field, such as medicine, agriculture, communication systems has increased the need to perform software testing in an effective and efficient manner so as to ensure the delivery of reliable and quality software. In the case of a component based software system, interactions among components are often complex and they may cause interaction errors. It is therefore important to check all the possible interactions among various components to uncover faults caused by their interactions. As each component may have multiple configurations, testing all possible combinations of components is practically impossible due to time and cost constraints. Furthermore, the number of test cases increases exponentially with the increase in number of components. A sampling strategy is therefore required to select a subset of configurations to be tested from the large interaction space. Combinatorial testing is a testing technique that samples the set of configurations in such a way that it covers all $t$-way ($t$ denotes the strength of testing) interactions of components [1].

Covering arrays (CAs) and mixed covering arrays (MCAs) are combinatorial structures that have enjoyed a wide range of application in the field of software and hardware testing [2]. Due to
the importance of CAs, significant research has
been carried out to construct CAs of optimal size
by the researchers in the past. A CA constructed to
perform t-way (2-way, 3-way, etc.) testing checks
only all t-way interactions of components. Empirical
studies show that a test set covering all
possible 2-way combinations of input parameter
values is effective for software systems [1, 3–5].
Dalal et al. [6] showed that testing all pair-wise
interactions in a software system finds a large
percentage of the existing faults. Kuhn et al. [7]
examined fault reports for many software systems
and concluded that more than 70% of the faults
are triggered by a 2-way interaction of the input
parameters. Faults can also be caused by the inter-
action of more than two parameters. In order to
uncover faults caused by the interaction of more
than two components, it is required to test higher
strength interactions of components. Empirical
studies in Kuhn et al. [7] and Kuhn and Reilly [8]
show that most of the faults are triggered by a
relatively low degree of interactions and suggest
the need to perform testing up to t = 6.

Consider a Graphical User Interface (GUI)
based on a windowing system which has five
components, each with three possible values as
shown in Table 1. For exhaustively testing the
components’ interactions in this system, 243 test
cases are required whereas only 11 test cases
for 2-way testing and 37 test cases for 3-way
testing are required respectively. Evidently, the
increase in strength of testing leads to the in-
crease in number of test cases. However, it is
quite often the case that certain components
have stronger interactions while others may have
few or none [9]. Hence, it is not desirable to
perform higher strength interaction testing of all
the components. A better way to test the system
is to identify the subsets of components which
are involved in stronger interactions and apply
higher strength interaction testing only on these
subsets to uncover the faults caused by their
interactions. This is achieved using the variable
strength covering array (VSCA), which is a CA
or MCA of fixed strength t and also contains a set
of disjoint CAs or MCAs of strength greater than

Assume, first four components have stronger in-

Figure 1. CA (11, 2, 3
5
)

Figure 2. VSCA (27; 2, 3
5
, (3, 3
4
))
Table 1. GUI based on a windowing system having five components, each with three values

<table>
<thead>
<tr>
<th>Kernel</th>
<th>Display Server (DS)</th>
<th>Window Manager (WM)</th>
<th>Display Server Communication Protocol (DSCP)</th>
<th>Graphical Interface (GI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linux</td>
<td>Weston</td>
<td>Awesome</td>
<td>X11</td>
<td>KDE Plasma</td>
</tr>
<tr>
<td>FreeBSD</td>
<td>KWin</td>
<td>Compiz</td>
<td>Wayland</td>
<td>Aqua</td>
</tr>
<tr>
<td>XNU</td>
<td>X.Org</td>
<td>OpenBox</td>
<td>Quartz</td>
<td>Gnome Shell</td>
</tr>
</tbody>
</table>

The problem of constructing an optimal VSCA is NP-complete [10, 11]. Although many algebraic and computational construction methods have been proposed by the researchers to construct optimal CA/MCA, fewer strategies (greedy and meta-heuristic) exist to construct optimal VSCA. The amount of work that has been done to construct VSCA using meta-heuristic techniques such as Simulated Annealing (SA), Particle Swarm Optimization (PSO), Harmony Search (HS) and their impressive results has motivated us to explore GA to construct optimal VSCA.

To exploit the strength of both greedy and meta-heuristic techniques we present a technique that augments GA with a greedy technique to construct optimal VSCA efficiently. Experiments are conducted to evaluate the performance of the proposed technique with the existing techniques.

However, the problem that exists with the construction of VSCA is the existence of constraints or dependencies between components values in terms of restrictions or compulsion on components values that can coexist. For instance, in the example shown in Table 1, Quartz is a Mac technology and therefore cannot be run on Linux or FreeBSD. This constraint must be taken into account when generating test cases so that Quartz and Linux/FreeBSD do not appear in the same test case. Similarly, KDE Plasma and XNU cannot appear in the same test case as XNU does not support KDE Plasma. If constraints and dependencies are considered, then combinatorial testing becomes constrained combinatorial testing. In this paper, we focus on combinatorial testing and leave constrained combinatorial testing for future work.

The remainder of this paper is organized as follows. Section 2 gives the necessary background on combinatorial objects. Section 3 gives an overview of GA. Section 4 presents various methods available to construct VSCA. Section 5 describes the proposed strategy to generate VSCA for t-way testing. Section 6 describes the implementation and presents result of experiments performed to compare the effectiveness of the proposed approach with other existing approaches. Section 7 presents threats to validity. Section 8 concludes the paper and future plans are outlined.

2. Background

This section discusses the necessary background related to combinatorial objects.

2.1. Orthogonal Array

An orthogonal array OA($N; t, k, v$) is an $N \times k$ array on $v$ symbols such that every $N \times t$ sub-array contains all ordered subsets of size $t$ from $v$ symbols exactly $\lambda$ times and they have the property $\lambda = N/v^t$ [12]. The use of OA in the field of software testing is limited due to the restrictions imposed on OA that all parameters have same number of values and that each pair of values can be covered the same number of times [13]. In general, OA is difficult to generate and its test suite is often quite large with $\lambda > 1$. However, OA has its advantages, such as making it relatively easy to identify the particular combination that caused a failure [11]. If an OA with $\lambda = 1$ exists for some value of $k$ and $v$, then it is an optimal array. To complement OA construction and to overcome its restrictions, CA and MCA have been introduced.
2.2. Covering Array

A covering array [12] denoted by $CA(N; t, k, v)$, is an $N \times k$ two dimensional array on $v$ symbols such that every $N \times t$ sub-array contains all ordered subsets from $v$ symbols of size $t$ at least $\lambda$ times. If $\lambda = 1$, it means that every $t$-tuple needs to be covered only once and we can use the notation $CA(N; t, k, v)$. Here, $k$ represents the number of values of each parameter and $t$ is the strength of testing. An optimal CA contains a minimum number of rows to satisfy the properties of the entire CA. The minimum number of rows is known as covering array number and is denoted by $CAN(t, k, v)$. A CA of size $N \times k$ represents a test set where each row corresponds to a test case, each column represents a component and the values in the column represent the domain of the respective component.

2.3. Mixed Covering Array

A mixed covering array [14], denoted by $MCA(N; t, k, v)$, is an $N \times k$ two dimensional array where $v_1, v_2, \ldots, v_k$ is a cardinality vector which indicates the values for every column. An MCA has the following two properties: i) Each column $i$ ($1 \leq i \leq k$) contains only elements from a set $S_i$ with $|S_i| = v_i$ and ii) The rows of each $N \times t$ sub-array cover all $t$-tuples of values from the $t$ columns at least once. The minimum $N$ for which there exists an MCA is called a mixed covering array number and is denoted by $MCAN(t, k, (v_1, v_2, \ldots, v_k))$. A shorthand notation can be used to represent MCAs by combining equal entries in $v_i : 1 \leq i \leq k$. An $MCA(N; t, k, (v_1, v_2, \ldots, v_k))$ can be represented as $MCA(N; t, k, (w^1, w^2, \ldots, w^s))$, where $k = \sum_{i=1}^{s} q_i$ and $w_j | 1 \leq j \leq s \subseteq \{v_1, v_2, \ldots, v_k\}$. Each element $w^j$ in the set $\{w^1, w^2, \ldots, w^s\}$ means that $q_i$ parameters can take $w^j$ values each. An MCA of size $N \times k$ represents a test set with $N$-test cases for a system with $k$ components, each with varying domain size.

2.4. Variable Strength Covering Array

A variable strength covering array [9], denoted by $VSCA(N; t, k, (v_1, v_2, \ldots, v_k), C)$, is an $N \times k$ CA or MCA of strength-$t$ containing $C$ where, $C$ is a set of disjoint CAs or MCAs each of strength greater than $t$. Each element of $C$ is a subset of VSCA and they can have variable strength of testing. For example, a $VSCA(N; 2, 4^35^6^2, \{CA(3, 4^3), MCA(4, 5^6^1)\})$ is shown in Fig. 3(a). Here, the overall array is an MCA having three components with four values, three with five values and two with six values each (the values of each component are labelled 0, 1, 2, 3, \ldots). It covers all 2-way interactions among components. In addition to this, it contains two sub arrays: a CA of strength-3 that covers all 3-way interactions among first three components with four values and an MCA of strength-4 covering all 4-way interactions among among three components with five values and one component with six values. The order of columns in the sub arrays in $C$ is important and they are listed consecutively from left to right. If a VSCA contains $n$ identical sub arrays with the same $t$, $k$ and $v$, they can be represented as $CA(t, v^k)^n$. For instance, $VSCA(N; 2, 3^{11}, (3, 3^4)^2)$ shown in Figure 3(b) represent a CA that covers all 2-way interactions among eleven components with three values and contains two disjoint sub arrays, each of which covers all 3-way interactions among four components with three values each.

3. Genetic Algorithm

The basics of GA were first proposed by Holland [15]. GA is a meta-heuristic search based optimization technique originating from the Darwinian theory of evolution by natural selection where fitter individuals are more likely to survive in a competing environment [16]. It is a global search technique characterized by evolution in every generation, starting with a randomly generated initial population. The initial population represents potential solutions to the given problem. Each individual in the population is associated with a fitness value that is calculated using a fitness function. The fitness function is a function of the objective that we want to optimize (maximize or minimize). The fitness value of an individual apprises us of the merit of a solution.
for the given problem. In each generation, the population evolves towards better solutions by means of evolutionary operators such as selection, crossover and mutation. This process continues until a satisfactory solution is found or the maximum number of generations is reached. As compared to other meta-heuristic techniques, GA starts with a population of solutions instead of a single solution that helps GA to cover the solution search space more thoroughly and avoid its chances of getting stuck in the local minima. Moreover, GA is easy to understand and can be applied to an optimization problem which can be described with chromosome encoding. On the contrary, the complexity of crossover and mutation operations is attributed to longer run time and, furthermore, GA cannot assure a constant optimization response time which limits its use in real time applications. The basic outline of GA is shown in Figure 4.

Having described the notations, in the next section we will briefly discuss the existing state-of-the-art algorithms for constructing optimal VSCA for pair-wise testing.

4. Related Work

In an extensive survey performed by Khalsa and Labiche in [17], it has been found that 75 tools/algorithms exist to generate CA/MCA for combinatorial testing but not all of them support construction of VSCA. The strategies that support the construction of VSCA are broadly classified into computational strategies and artificial intelligence based strategies. Computational strategies use greedy approach to construct VSCA by using either one-test-at-a-time or one-parameter-at-a-time approach. The strategies based on one-test-at-a-time approach use a greedy heuristic and try to select a test case that covers the maximum number of uncovered interactions from a pool of candidate test cases. However, selecting such a test case itself is an NP-complete problem [18]. Some of the strategies that use one-test-at-a-time approach are the Test Vector Generator (TVG) [19], Pairwise Independent Combinatorial Testing (PICT) [20], Intelligent Test Case Handler1 (ITCH), Density [21], DA-RO and DA-FO [22], and TSG [23]. The strategies that use one-parameter-at-a-time approach are ACTS [24,25] and ParaOrder [21].

Recently the search based software testing (SBST) is increasingly gaining importance and is been used to solve a wide range of problems in software testing. Meta-heuristic techniques are being used by the SBST community to find an optimal solution for software testing problems. The problem of generating an optimal CA/MCA/VSCA is also considered as a SBST problem [26,27]. Meta-heuristic techniques start by searching over a large set of feasible solutions and can often find better solutions with

\[ \begin{array}{cccccccc}
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 2 & 3 & 1 \\
1 & 2 & 3 & 1 & 4 & 3 & 0 & 1 \\
2 & 3 & 2 & 4 & 2 & 3 & 2 & 2 \\
0 & 2 & 1 & 2 & 3 & 0 & 1 & 5 \\
\end{array} \]

\[ \begin{array}{cccccccc}
3 & 0 & 0 & 3 & 4 & 5 & 3 \\
0 & 3 & 3 & 3 & 2 & 2 & 4 & 4 \\
\end{array} \]

\[ \begin{array}{cccccccc}
0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 2 \\
2 & 1 & 2 & 0 & 0 & 1 & 0 & 1 \\
1 & 2 & 1 & 2 & 1 & 2 & 2 & 0 \\
0 & 2 & 1 & 2 & 1 & 2 & 1 & 1 \\
\end{array} \]

\[ \begin{array}{cccccccc}
0 & 1 & 0 & 0 & 0 & 1 & 2 & 1 \\
1 & 1 & 0 & 2 & 1 & 1 & 1 & 2 \\
2 & 2 & 1 & 0 & 0 & 1 & 2 & 2 \\
\end{array} \]

Figure 3. Representation of VSCA

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1 IBM Intelligent Test Case Handler
generate an initial population $P$ randomly
evaluate fitness of each individual in $P$ using the fitness function
while (\( \text{generation} \leq \text{maximum generation} \) and solution not found)
    select a subset of individuals $P'$ from current generation for offspring production
    apply crossover to $P'$
    apply mutation to $P'$
    replace low fitness individuals in $P$ by offspring in $P'$
evaluate $P$
end while
return individual with highest fitness

Figure 4. Outline of basic GA

fewer computational efforts as compared to other algorithms, iterative methods or simple heuristics [28]. To the best of our knowledge, only five meta-heuristic based strategies to generate VSCA exist in the literature. Table 2 list features of all tools/algorithms that use meta-heuristic techniques and some selected tools that use greedy techniques to construct CA/MCA/VSCA.

5. The Proposed Approach for Construction of VSCA

In this section, we present our proposed strategy of VSCA-GA that aims to generate an optimal VSCA to cover all (100%) $t$-way and $t_i$-way interactions between components in a component based system. Here, $t$ denotes the overall strength of VSCA and $t_i$ denotes the strength of sub arrays. VSCA-GA uses a greedy based approach to GA to generate optimal VSCA. Let $\text{VSCA}(N; t, k, (v_1, v_2, \ldots, v_k), C)$ represent a VSCA configuration. VSCA-GA starts by creating an initial population of $P_{\text{size}}$ individuals where each individual chromosome represents a candidate solution which is a VSCA of size $N \times k$. Here, $N$ the number of rows of VSCA corresponds to test cases and $k$ represents the number of components in a component based system. At the start of the search process $N$ is unknown, so we use the method suggested by Stardom [39], where we start with a large random array and apply binary search repeatedly until a solution is found. In case the size of $N$ is known in advance, i.e. best bound achieved in the existing state-of-the-art, we can start with the known size and try to optimize it further.

An individual chromosome in the population is represented by $\text{VSCA}_f|1 \leq f \leq P_{\text{size}}$ and each $\text{VSCA}_f$ in the population has a fitness associated with it which is defined as the total number of distinct variable strength interactions covered by it. It is calculated as

$$\text{Fitness}(\text{VSCA}_f) = \sum_{i=t, t_1, \ldots, t_n} \text{number of distinct } i\text{-way interactions covered by } \text{VSCA}_f \quad (1)$$

Where, $t$ is the overall strength of VSCA, $t_1, t_2, \ldots, t_n$ are the strength of sub-arrays.

After initialization, GA searches the solution space by applying genetic operators such as selection, crossover and mutation repeatedly to find the best solution. The process continues until a solution is found or the maximum number of iterations is reached. In case VSCA-GA starts by taking $N$ from the existing literature and a solution is found at this $N$, then the size of VSCA is decreased by one, otherwise the size of VSCA is increased by one and VSCA-GA is executed again in both cases. When VSCA-GA is re-executed, we seed the initial population by supplying the best VSCA generated in the previous run. If the size of VSCA in the current run is less than that in the previous run, we decrease the size of seeded VSCA by one by removing the test case that contributes least to the fitness of VSCA whereas, if the size of VSCA in the current run is greater than the size in the previous run, then we add a randomly generated test case to the existing VSCA. The various steps of VSCA-GA strategy are explained below.
Table 2. Comparison of various tools/algorithms for constructing CA/MCA/VSCA for CIT

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Tool / Algorithm</th>
<th>Variable Strength</th>
<th>Maximum Strength</th>
<th>Technique Employed</th>
<th>Test Generation Strategy</th>
<th>Constraint Handling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AETG [1]</td>
<td>x</td>
<td>2</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>2</td>
<td>ITCH [1]</td>
<td>✔</td>
<td>6</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>3</td>
<td>TVG [19]</td>
<td>✔</td>
<td>6</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>4</td>
<td>PICT [20]</td>
<td>✔</td>
<td>6</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>5</td>
<td>Density [21]</td>
<td>✔</td>
<td>3</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>6</td>
<td>DA-RO [22]</td>
<td>✔</td>
<td>3</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>7</td>
<td>DA-FO [22]</td>
<td>✔</td>
<td>3</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>8</td>
<td>TSG [23]</td>
<td>✔</td>
<td>3</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>9</td>
<td>Jenny [29]</td>
<td>x</td>
<td>8</td>
<td></td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>10</td>
<td>ACTS (IPOG) [24, 25]</td>
<td>✔</td>
<td>6</td>
<td>Greedy One Test at a Time</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>11</td>
<td>ParaOrder [21]</td>
<td>✔</td>
<td>3</td>
<td></td>
<td>One Parameter at a Time</td>
<td>✔</td>
</tr>
<tr>
<td>12</td>
<td>SA [9]</td>
<td>✔</td>
<td>3</td>
<td>Simulated Annealing</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>13</td>
<td>GA [30]</td>
<td>x</td>
<td>3</td>
<td>Genetic Algorithm</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>14</td>
<td>ACA [30]</td>
<td>x</td>
<td>3</td>
<td>Ant Colony Optimiz</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>15</td>
<td>ACS [31]</td>
<td>✔</td>
<td>3</td>
<td>Ant Colony Optimiz</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>16</td>
<td>TSA [32]</td>
<td>x</td>
<td>6</td>
<td>Tabu Search</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>17</td>
<td>GAPT [33]</td>
<td>x</td>
<td>2</td>
<td>Genetic Algorithm</td>
<td>One Test at a Time</td>
<td>✔</td>
</tr>
<tr>
<td>18</td>
<td>PwiseGen [34]</td>
<td>x</td>
<td>2</td>
<td>Meta-heuristic</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>19</td>
<td>VS-PSTG [35]</td>
<td>✔</td>
<td>6</td>
<td>Particle Swarm Optimization</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>20</td>
<td>HSS [36]</td>
<td>✔</td>
<td>15</td>
<td>Harmony Search</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>21</td>
<td>HSTCG [37]</td>
<td>✔</td>
<td>7</td>
<td>Harmony Search</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>22</td>
<td>CASA [27]</td>
<td>x</td>
<td>3</td>
<td>Simulated Annealing</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>23</td>
<td>PSO [38]</td>
<td>x</td>
<td>2</td>
<td>Particle Swarm Optimization One Parameter at a Time</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>24</td>
<td>PSO [38]</td>
<td>x</td>
<td>2</td>
<td>Particle Swarm Optimization</td>
<td></td>
<td>✔</td>
</tr>
</tbody>
</table>

5.1. A Greedy Approach to Generate Initial Population

When GA is used to construct VSCA, the role of initial population on the performance of GA cannot be ignored as it can affect the convergence speed and quality of the final solution [40, 41]. Generally, initial population is generated randomly. However, recognizing the effect of initial population on GA performance, several population initialization methods for GA have been proposed in the past by the researchers [41–46]. Here, we present a greedy approach for generating a good quality initial population of VSCA, which is achieved by focusing on the coverage of maximum number of possible uncovered interactions.

Let us consider the system under test consisting of \( k \) components where a component is represented by \( C_m \) for \( 1 \leq m \leq k \) and each component \( C_m \) can take values from 0 to \( (v_m - 1) \) (\( v_m \) is the number of possible values of component \( C_m \)). The \( j^{th} \) component of \( C_m \) represented by \( \text{val}_{mj} \). To generate an initial population of VSCAs, VSCA-GA starts by computing and storing all the possible \( t \)-way and \( t_i \)-way interactions between the values of all the components in an interaction list \( L \), based on the configuration of VSCA. Then, it calculates the number of uncovered interactions of each value \( \text{val}_{mj} \) of every component \( C_m \), stores it in a variable \( N_{\text{uncovered}}(\text{val}_{mj}) \) and assigns a probability of selection denoted by \( P(\text{val}_{mj}) \) to each of them. The probability of selection assigned to a value \( \text{val}_{mj} \) of component \( C_m \) represents its chances of getting selected when a test case is created. In our case, the probability assigned to a value \( \text{val}_{mj} \) of component \( C_m \) depends upon the number of uncovered interactions of \( \text{val}_{mj} \) as well as the total number of uncovered interactions of component \( C_m \). Initially all values \( \text{val}_{mj} \) of a component \( C_m \) are involved in an equal number of uncovered
interactions, therefore each of them will have an equal probability of getting selected. For instance, if a component has four possible values then initially each one of them will have the probability of selection equal to 0.25. VSCA-GA updates the interaction list $L$ by eliminating interactions that are covered in $tc_{f1}$. Let the value $val_{ms}$ of component $C_m$ be selected in $tc_{f1}$ and the number of interactions covered by $val_{ms}$ in $tc_{f1}$ be $N_{covered}(val_{ms})$, then the number of interactions of $val_{ms}$ left uncovered is denoted by $N'_{uncovered}(val_{ms})$ and is calculated as:

$$N'_ {uncovered}(val_{ms}) = N_{uncovered}(val_{ms}) - N_{covered}(val_{ms}) \quad (2)$$

Let $P_{old}(val_{ms})$ denote the probability of selection of value $val_{ms}$ before selection, then after selection the probability of $val_{ms}$ becomes:

$$P_{new}(val_{ms}) = P_{old}(val_{ms}) \times \frac{N'_ {uncovered}(val_{ms})}{N_{uncovered}(val_{ms})} \quad (3)$$

The decrease in the probability of value $val_{ms}$ is calculated using Equation 4 and is distributed among the remaining values $val_{mj} \mid 1 \leq j \leq v_m$ and $j \neq s$ of component $C_m$ according to Equation 5.

$$P_{decrement}(val_{ms}) = P_{old}(val_{ms}) \times \left(1 - \frac{N'_ {uncovered}(val_{ms})}{N_{uncovered}(val_{ms})}\right) \quad (4)$$

$$P_{new}(val_{mj}) = P_{old}(val_{mj}) + \left(\frac{N_{uncovered}(val_{mj})}{\sum_{j=1}^{v_m} N_{uncovered}(val_{mj})} \times \frac{P_{decrement}(val_{ms})}{\sum_{j=1}^{v_m} P_{decrement}(val_{ms})}\right) \quad (5)$$

Equation 5 increases the probability of the value $val_{mj}$ of component $C_m$ based on the number of its uncovered interactions and the total number of uncovered interactions of the remaining values (except $val_{ms}$) of component $C_m$. Hence, the higher the number of remaining uncovered interactions of a value, the higher will be the increase in its probability and vice versa, thereby getting greedy by extending a higher opportunity of selection to the values with maximum uncovered interactions. Once the probability of each value of every component is updated, the number of uncovered interactions of the selected value $val_{mj} \forall m$ is updated by assigning the value of $N'_ {uncovered}(val_{ms})$ to $N_{uncovered}(val_{ms})$. The succeeding test cases $tc_{f2}[2 \leq i \leq N]$ are generated by selecting a value for each component based on the probabilities that are updated after the generation of every test case. Since each value $val_{mj}$ of a component $C_m$ may have different probability of selection, to select a value of a component, a random number is generated in the range $[0,1]$ and based on the interval in which the random number falls; the value $val_{mj}$ of the component $C_m$ is selected. For instance, consider a component $C_m$ having four possible values and assume that at some point of time during the test case generation process, the probability of selection of each of the four values $val_{m1}$, $val_{m2}$, $val_{m3}$ and $val_{m4}$ becomes 0.20, 0.35, 0.35 and 0.10 respectively. If the generated random number lies in the range $[0,0.2]$ then value $val_{m1}$ is selected, if it lies in the range $(0.2,0.55]$ then value $val_{m2}$ is selected, if it lies in the range $(0.55,0.9]$ then value $val_{m3}$ is selected otherwise $val_{m4}$ is selected. An example to illustrate the greedy approach to generate initial population is given below.

Example: Let us consider a component based a system having configuration $(N; 2, 2^3 3^1, CA(3, 2^3))$ as shown below:

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>a2</td>
<td>a3</td>
<td>a4</td>
</tr>
<tr>
<td>b1</td>
<td>b2</td>
<td>b3</td>
<td>b4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>c4</td>
</tr>
</tbody>
</table>

To construct a VSQA in the initial population, VSCA-GA assigns a probability of selection to each value of every component. Initially, each value of a component has an equal number of uncovered interactions; therefore each of them will have an equal probability of selection. The
probability of selection of each value of every component is shown below:

\[
\begin{array}{cccc}
C_1 & C_2 & C_3 & C_4 \\
P(a1)=0.5 & P(a2)=0.5 & P(a3)=0.5 & P(a4)=0.333 \\
P(b1)=0.5 & P(b2)=0.5 & P(b3)=0.5 & P(b4)=0.333 \\
& & & P(c4)=0.333 \\
\end{array}
\]

The first test case TC_1 is constructed by selecting a value for each component randomly from their respective input domain. Let TC_1 be: a1, b2, a3, b4.

Now, VSCA-GA changes the probability of selection of each value of every component based on the number of their uncovered interactions using Equations 2–5. The new probabilities become:

\[
\begin{array}{cccc}
C_1 & C_2 & C_3 & C_4 \\
P(a1)=0.32 & P(a2)=0.68 & P(a3)=0.32 & P(a4)=0.166 \\
P(b1)=0.68 & P(b2)=0.32 & P(b3)=0.68 & P(b4)=0.417 \\
& & & P(c4)=0.417 \\
\end{array}
\]

Subsequently, for generating the next test case TC_2, VSCA-GA generates random numbers. Let the random numbers generated be 0.2, 0.5, 0.2 and 0.3 for each component respectively. Therefore, TC_2 will be: a1, a2, a3, b4.

Now, VSCA-GA again changes the probability of selection of each value of every component based on the number of their uncovered interactions using Equations 2–5. The new probabilities become:

\[
\begin{array}{cccc}
C_1 & C_2 & C_3 & C_4 \\
P(a1)=0.18 & P(a2)=0.43 & P(a3)=0.18 & P(a4)=0.235 \\
P(b1)=0.82 & P(b2)=0.57 & P(b3)=0.82 & P(b4)=0.209 \\
& & & P(c4)=0.556 \\
\end{array}
\]

The same procedure is repeated to construct the remaining \((N-2)\)-test cases. Once a VSCA is generated, the same procedure is repeated to generate all the remaining VSCAs in the initial population. Notably, every time a new VSCA is generated, the interaction list L is reinitialized to store all the possible \(t\)-way and \(t_i\)-way interactions between the values of all the components based on the configuration of VSCA.

### 5.2. A Greedy Approach to Perform Crossover

The next step after initialization is the application of selection, crossover and mutation operators repeatedly to generate optimal VSCA that covers all possible \(t\)-way and \(t_i\)-way interactions. The crossover operator combines the genes of two or more parents to generate an offspring. It is based on the idea that the exchange of information between good chromosomes will generate even better offspring [47]. There are many variations of the crossover method, namely single-point crossover, two-point crossover, multi-point crossover, uniform crossover, etc. The number of crossover points determines how many segments are exchanged between the parents. The length (number of genes) of each segment may vary and it depends on the position of crossover points. VSCA-GA performs a crossover at the boundaries of test cases and the length of a segment is always equal to one (i.e. one test case). When a crossover is performed, it is quite possible that during the exchange of information between parents, some good features of a parent may get lost. In our case, based on the configuration of VSCA each test case \(tc_{fi}|1 \leq i \leq N\) in VSCA_f covers some fixed number of \(t\)-way and \(t_i\)-way interactions, out of which some interactions are distinctly covered by \(tc_{fi}\) only. When a random crossover is performed, it may happen that during the exchange of information between two parent VSCAs say VSCA_1 and VSCA_2, the best test case \(tc_{f1}\) covering maximum number of distinct interactions in VSCA_1 may get exchanged with the test case \(tc_{2i}\) of VSCA_2. This may result in the gain of new interactions as well as loss of existing distinct interactions covered by \(tc_{f1}\) in VSCA_1 thereby, reducing the net gain in fitness after the crossover. The net gain in fitness is calculated using Equation 6.

\[
\text{Net gain in fitness(VSCA}_f) = \frac{\text{Number of new interactions gained}}{\text{Number of existing distinct interactions lost}} - 1
\]
In order to minimize the loss of existing distinct interactions and to maximize the net gain in fitness during a crossover, VSCA-GA uses a greedy approach to perform a crossover. It takes the number of test cases which are to be exchanged during the crossover as input (NTC) instead of the number of crossover points, which helps it in selecting the test cases greedily for crossover. VSCA-GA starts by selecting VSCAs using roulette wheel selection to become parents during the crossover. In the roulette wheel selection, a probability is being assigned to each individual in the population. This probability is calculated on the basis of the fitness of the individual and thus the individuals with higher fitness have better chances of getting selected for reproduction. Out of the two parents selected using roulette wheel for crossover, VSCA-GA chooses a parent with higher fitness. Let the higher fitness parent be parent\(_1\) then VSCA-GA calculates the number of distinct t-way and t\(_i\)-way interactions covered by each test case of parent\(_1\). Subsequently, it checks whether the number of test cases to be exchanged (NTC) is equal to the number of test cases covering the least number of distinct interactions. There are three possibilities:

1. The number of test cases covering the least number of distinct interactions is equal to NTC – In this case VSCA-GA performs crossover by exchanging the test cases that cover the least number of distinct interactions in parent\(_1\) by the respective test cases of parent\(_2\). For instance, consider a system \(A\) having configuration \((N; 2, 3^5, CA(3, 3^4))\) (the value of each component is labelled 0, 1, 2) and let \(N\) be 7 which means that the VSCA will consist of 7 test cases represented by TC\(_1\), TC\(_2\), ..., TC\(_7\). Each test case TC\(_i\)\(1 \leq i \leq 7\) contains a value 0/1/2 corresponding to each component. Let NTC be 2. After calculating the number of distinct interactions covered by each of the 7 test cases in parent\(_1\), it has been found that two test cases TC\(_2\) and TC\(_5\) cover the least number of distinct interactions (i.e., 4) in parent\(_1\). Hence, the number of test cases covering the least number of distinct interactions in parent\(_1\) is equal to NTC. Accordingly, a crossover is performed by exchanging TC\(_2\) and TC\(_5\) in parent\(_1\) with the respective test cases TC\(_2\) and TC\(_5\) of parent \(_2\) as shown in Figure 5(a).

2. NTC is greater than the number of test cases covering least number of distinct interactions. Here VSCA-GA selects first NTC test cases in parent\(_1\) when sorted in the ascending order by the number of distinct interactions covered by them and applies a crossover at these positions. For instance, in the aforementioned system \(A\), let NTC be 3. Here, NTC is greater than the number of test cases covering the least number of interactions, so the crossover is performed by exchanging TC\(_2\), TC\(_5\) and TC\(_4\) (which covers next least number of interactions i.e. 7 after TC\(_2\) and TC\(_5\)) with the respective test cases of parent\(_2\) as shown in Figure 5(b).

3. The number of test cases covering the least number of distinct interactions is greater than NTC: Here VSCA-GA calculates all the t-way and t\(_i\)-way interactions covered by the respective test cases of parent\(_2\) and performs crossover by exchanging test cases that cover the maximum number of interactions not covered by parent\(_1\). Again, for the aforementioned system \(A\), two test cases TC\(_2\) and TC\(_5\) in parent\(_1\) cover the least number of distinct interactions (i.e., 4). Let NTC be 1, which is less than the number of test cases covering the least number of distinct interactions in parent\(_1\). In this case, VSCA-GA calculates the number of interactions covered by the respective test cases of parent\(_2\), in our case TC\(_2\) and TC\(_5\). It is clear from Figure 5(c), that TC\(_3\) covers 5 interactions as compared to TC\(_2\) which covers only 1 interaction, not covered in parent\(_1\). Hence, our strategy performs a crossover by exchanging test cases TC\(_5\) in parent\(_1\) and parent\(_2\). By choosing the test case in parent\(_1\) that covers the least number of distinct interactions and exchanging it with a test case of parent\(_2\) that covers maximum number of interactions not covered by parent\(_1\), we ensure that the resulting offspring is of better quality than its parent by minimizing the loss of existing interactions and maximizing the gain.
5.3. A Greedy Approach to Perform Mutation

Mutation has a significant effect on the performance of GA as the mutation operator randomly modifies, with a given probability, one or more genes of a chromosome, thus increasing the diversity of the population and avoids getting stuck in the local minima. In traditional GA, every individual has an equal probability of getting mutated irrespective of their fitness [48]. Thus the probability of an individual with the highest fitness to be disrupted by a mutation is equal compared to the one with the lowest fitness. Hence a mutation strategy is needed to mutate an individual to maximize improvement in fitness by minimizing fitness loss due to the mutation. Here, we present a greedy mutation strategy to perform a mutation. First, we select an individual VSCA for a mutation and list all the t-way and t_i-way interactions left uncovered by the selected individual. Subsequently starting from the highest strength (t_h) uncovered interaction, we check interactions of strength t_h that occurs multiple times in VSCA and replace one of its occurrences with the uncovered t_h interaction in an attempt to increase its overall fitness. However, when an existing interaction is replaced with an uncovered interaction, then in addition to the gain of new interactions some old distinct interactions may get lost. Hence, to maximize the net gain after mutation, we calculate the number of distinct interactions covered by the multiple occurring interactions in the respective test cases and replaces the one which covers the least number of distinct interactions. In case more than one test case covers the least number of distinct interactions we replace the one
which covers the least number of higher strength interactions. For instance, consider a system having configuration \((N; 2,2^43^2, MCA(3, 2^23^2))\) as shown in Figure 6. It is evident from Figure 6 that the VSCA selected for mutation does not cover the triplet ‘b3 b4 a6’. When examining the VSCA, it is found that the triplet ‘b3 a4 b6’ is covered by both TC5 and TC6. Hence, one occurrence of ‘b3 a4 b6’ can be replaced by ‘b3 b4 a6’. To replace an occurrence of ‘b3 a4 b6’ by ‘b3 b4 a6’, the proposed greedy approach to mutation calculates the total number of distinct interactions of strength-2 and strength-3 covered by b3, a4 and b6 in TC5 and TC6. In our case both TC5 and TC6 cover an equal number of distinct interactions, so the proposed approach replaces ‘b3 a4 b6’ in TC5 which covers a smaller number of distinct interactions of higher strength ‘3’ by the uncovered triplet ‘b3 b4 a6’.

The overall VSCA-GA strategy can be found in Appendix.

6. Experimental Results

To assess the effectiveness of VSCA-GA strategy, we implemented the proposed strategy by extending an open-source tool PWiseGen [49]. It is an open-source tool written in Java to generate pair-wise (2-way) test set using GA. It does not provide support for the construction of CA of strength \(t > 2\) as well as VSCA construction. We have extended PWiseGen by adding the capability to generate VSCA of strength up to 6 using the greedy strategies proposed in Section 5 to generate the initial population, crossover and mutation. We call it PWiseGen-VSCA.

To compare the performance of PWiseGen-VSCA with the existing greedy based strategies such as IPOG, PICT, ITCH, TVG, DA-RO, DA-FO, ParaOrder, TSG and AI based strategies such as SA, ACS, VS-PSTG, HSS and HSTCG based on VSCA size, we performed experiments on a set of four benchmark problems taken from Cohen et al. [9], Ahmed et al. [35] and Alsewari and Zamli [36]. As the VSCA size is not dependent on the execution environment, we compare our result directly with the results published in literature [9, 21–23, 31, 35, 36] with respect to VSCA size.

The results of experiments conducted to compare VSCA size on four VSCA configurations with various sub-configuration settings are shown in Table 3, Table 4, Table 5 and Table 6 respectively. Cells marked NA (not available) in the table signify that the results are not available in the publications and the cells marked ‘–’ signify that the tool/algorithm does not support the specified strength. As VSCA-GA produces non-deterministic results, we ran each configuration 30 times on PWiseGen-VSCA and reported the best VSCA size obtained over 30 runs. It can be observed from Table 3, Table 4, Table 5 and Table 6 that AI-based strategies generally perform better than their greedy counterparts.

When AI-based strategies are compared to each other, we can see that SA and ACS support construction of VSCA of strength \(t \leq 3\) ...
### Table 3. VSCA Size for VSCA configuration VSCA\((N; 2, 3^{15}, C)\)

<table>
<thead>
<tr>
<th>{C}</th>
<th>No. of interactions</th>
<th>PICT</th>
<th>ITCH</th>
<th>DA-RO</th>
<th>DA-FO</th>
<th>Para</th>
<th>Order</th>
<th>TVG</th>
<th>TSG</th>
<th>IPOG</th>
<th>SA</th>
<th>ACS</th>
<th>VS-PSTG</th>
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<th>PWiseGen-VSCA</th>
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<tbody>
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<td>152</td>
<td>132</td>
<td>170</td>
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<td>82</td>
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### Table 4. VSCA Size for VSCA configuration VSCA\((N; 2, 3^{20}10^2, C)\)

<table>
<thead>
<tr>
<th>{C}</th>
<th>No. of interactions</th>
<th>PICT</th>
<th>ITCH</th>
<th>DA-RO</th>
<th>DA-FO</th>
<th>Para</th>
<th>Order</th>
<th>TVG</th>
<th>TSG</th>
<th>IPOG</th>
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<th>PWiseGen-VSCA</th>
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<tbody>
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### Table 5. VSCA Size for VSCA configuration VSCA\((N; 2, 4^35^36^2, C)\)

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Table 6. VSCA Size for VSCA configuration VSCA($N; 2, 10^19 \times 1, 7 \times 1, 6 \times 1, 5 \times 4 \times 1, 2, C$)

<table>
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<tr>
<th>$\phi$</th>
<th>No. of interactions</th>
<th>PICT</th>
<th>ITCH</th>
<th>Density</th>
<th>Para TVG</th>
<th>IPOG</th>
<th>SA</th>
<th>ACS</th>
<th>VS-PSTG</th>
<th>HSS</th>
<th>PWiseGen-VSCA</th>
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<tr>
<td>MCA(3, $10^19 \times 1$)</td>
<td>1266</td>
<td>102</td>
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<td>99</td>
<td>90</td>
<td>NA</td>
<td>NA</td>
<td>97</td>
<td>94</td>
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<td>3125</td>
<td>765</td>
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<td>2397</td>
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<td>124157</td>
<td>2276</td>
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<td>NA</td>
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<td>MCA(6, $7^1 \times 5^1 \times 1^3 \times 1^2$)</td>
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<td>5157</td>
<td>NA</td>
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Table 7. VSCA generation time (in seconds) for VSCA configuration VSCA($N; 2, 31^1$, C)

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>IPOG</th>
<th>TVG</th>
<th>PWiseGen-VSCA</th>
</tr>
</thead>
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<tr>
<td>$\phi$</td>
<td>0.077</td>
<td>0.056</td>
<td>2.976</td>
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<tr>
<td>CA(3, $3^3$)</td>
<td>0.009</td>
<td>0.071</td>
<td>1.32</td>
</tr>
<tr>
<td>CA(3, $3^2$)</td>
<td>0.025</td>
<td>0.062</td>
<td>13.5</td>
</tr>
<tr>
<td>CA(3, $3^3$)</td>
<td>0.023</td>
<td>0.076</td>
<td>5.424</td>
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<tr>
<td>CA(3, $3^4$)</td>
<td>0.012</td>
<td>0.088</td>
<td>60.042</td>
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<td>CA(3, $3^5$)</td>
<td>0.03</td>
<td>0.098</td>
<td>11.4</td>
</tr>
<tr>
<td>CA(3, $3^6$)</td>
<td>0.013</td>
<td>0.141</td>
<td>48.06</td>
</tr>
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<td>CA(3, $3^7$)</td>
<td>0.023</td>
<td>0.161</td>
<td>57.6</td>
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<td>CA(3, $3^8$)</td>
<td>0.019</td>
<td>0.204</td>
<td>97.8</td>
</tr>
<tr>
<td>CA(3, $3^9$)</td>
<td>0.048</td>
<td>2.008</td>
<td>211.08</td>
</tr>
<tr>
<td>CA(3, $3^10$)</td>
<td>0.008</td>
<td>0.302</td>
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<td>CA(3, $3^11$)</td>
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<td>0.108</td>
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<td>0.011</td>
<td>0.189</td>
<td>5431.8</td>
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<td>0.862</td>
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<td>CA(3, $3^14$)</td>
<td>0.015</td>
<td>0.499</td>
<td>6.6</td>
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<td>CA(3, $3^15$)</td>
<td>0.046</td>
<td>3.853</td>
<td>12035.4</td>
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<td>CA(3, $3^16$)</td>
<td>0.093</td>
<td>1.388</td>
<td>19.44</td>
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<td>CA(3, $3^17$)</td>
<td>0.078</td>
<td>11.685</td>
<td>21183.6</td>
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Table 9. VSCA generation time (in seconds) for VSCA configuration VSCA($N; 2, 41^5 \times 6^2$, C)

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>IPOG</th>
<th>TVG</th>
<th>PWiseGen-VSCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.002</td>
<td>0.035</td>
<td>2.37</td>
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<tr>
<td>CA(3, $4^1$)</td>
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<td>0.041</td>
<td>2.106</td>
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<td>0.002</td>
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<td>433.8</td>
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<tr>
<td>CA(3, $4^3$)</td>
<td>0.005</td>
<td>0.077</td>
<td>0.39</td>
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<td>CA(3, $4^4$)</td>
<td>0.001</td>
<td>0.082</td>
<td>0.577</td>
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<tr>
<td>CA(3, $4^5$)</td>
<td>0.016</td>
<td>0.189</td>
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<td>CA(3, $4^6$)</td>
<td>0.001</td>
<td>0.082</td>
<td>52.6344</td>
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<td>CA(3, $4^7$)</td>
<td>0.032</td>
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<td>CA(3, $4^8$)</td>
<td>0.023</td>
<td>0.917</td>
<td>6992.4</td>
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<td>0.089</td>
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<td>CA(3, $4^{11}$)</td>
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<td>CA(5, $4^5$)</td>
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<td>2.84</td>
<td>30.6</td>
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<td>CA(5, $4^6$)</td>
<td>0.296</td>
<td>26.193</td>
<td>7485.6</td>
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Table 10. VSCA generation time (in seconds) for VSCA configuration VSCA($N; 2, 10^19 \times 1, 7 \times 1, 6 \times 1, 5 \times 4 \times 1, 2, C$)

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>IPOG</th>
<th>TVG</th>
<th>PWiseGen-VSCA</th>
</tr>
</thead>
<tbody>
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<td>0.414</td>
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<td>CA(3, $10^19 \times 1$)</td>
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<td>0.865</td>
<td>5.148</td>
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<td>CA(3, $7 \times 1$)</td>
<td>0.007</td>
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<td>6.72</td>
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<td>CA(3, $4^3$)</td>
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<td>0.131</td>
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<td>0.044</td>
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<td>CA(3, $4^2$)</td>
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<td>6.0684</td>
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<td>0.021</td>
<td>635.22</td>
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<td>CA(3, $4^5$)</td>
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<td>7.504</td>
<td>85.8</td>
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<tr>
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<td>0.733</td>
<td>38.548</td>
<td>484.02</td>
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only whereas VS-ПSTG supports construction of VSCA of strength up to 6. The published results [36] show that unlike other greedy and AI-based strategies, HSS support construction of VSCA of strength up to 15 but nothing is mentioned about the efficiency of HSS in terms of VSCA generation time. From Table 3, we can infer that PWiseGen-VSCA outperforms ACS whereas the results in Table 4 and Table 5 are comparable. Although PWiseGen-VSCA supports construction of higher interaction strength VSCA however, VSCA generation time increases with the increase in interaction strength which makes it infeasible to generate higher strength VSCAs. It is evident from Tables 3-6 that PWiseGen-VSCA generates better results as compared to VS-PSTG, HSTCG and HSS. From Tables 3–6, it is clear that SA outperforms existing state-of-the-art strategies for lower interaction strength \((t \leq 3)\), however, the results generated by PWiseGen-VSCA are equal or close to SA.

Finally from Tables 3–6, we can conclude that PWiseGen-VSCA generates optimal VSCA most of the time as compared to other greedy and meta-heuristic techniques for strength \(t \leq 6\).

It is difficult to compare PWiseGen-VSCA with the existing state-of-the-art algorithms in terms of VSCA generation time, as the generation time is dependent on the running environment and most of the algorithm implementations are not publicly available. To perform a fair comparison, we restrict the comparison of VSCA generation time against publicly available algorithm implementation: ACTS (IPOG) and TVG. These tools are run on Windows using an INTEL Pentium Dual Core 1.73 GHZ processor with 1.00 GB of memory. The results of comparison made on the dataset of Tables 3–6 with respect to VSCA generation time (in seconds) are shown in Tables 7–10 respectively. It is evident from Tables 7–10 that PWiseGen-VSCA requires more time to construct VSCA as compared to ACTS (IPOG) and TVG, however, the extra time consumed by PWiseGen-VSCA allowed the construction of VSCAs of smaller size.

7. Threats to Validity

One important threat to validity of the effectiveness of our approach is that we could not use any sophisticated statistical hypothesis tests such as Welch’s \(t\)-test to assess and compare PWiseGen-VSCA with the existing meta-heuristic techniques for constructing VSCA as we do not have access to the source code of any of them. Also, we could not compare the efficiency of PWiseGen-VSCA in terms of VSCA generation time with the existing meta-heuristic techniques because of the above mentioned reason.

8. Conclusion and Future Work

In this paper we have presented and evaluated VSCA-GA, a strategy based on GA to construct optimal VSCA for \(t\)-way testing. The strategy is implemented in PWiseGen-VSCA. Our strategy exploits the strength of both greedy and meta-heuristic techniques by integrating greedy technique with GA. The experiments conducted on a set of benchmark problems show that PWiseGen-VSCA outperforms the existing state-of-the-art algorithms except SA in terms of VSCA sizes. However, our results are comparable to SA which generates VSCA for strength \(t\) up to 3 whereas VSCA-GA constructs VSCA for strength \(t\) up to 6.

In future, we plan to construct VSCA to handle feature constraints and try to improve the efficiency of PWiseGen-VSCA to construct higher strength VSCA.

References


Appendix

Input: VSCA configuration: \((t, k, (v_1, v_2, \ldots, v_k), C)\), VSCA size: \(N\), Population Size: \(P_{\text{size}}\), Maximum Number of Generations: \(NOG\), Number of Reproductions: \(NOR\), Number of Test Cases for Crossover: \(NTC\)

Output: Optimal VSCA

Algorithm 1. VSCA-GA

1. procedure VSCA-GA
2. Initialize \(G := 0\) \(\triangleright \) generation number
3. \(\triangleright \) Generate initial population \(pop_1\)
4. for each VSCA \(f\) in \(pop_1\) do \(\triangleright 1 \leq f \leq P_{\text{size}}\)
5. Create an interaction list \(L\) of all \(t\)-way and \(t_i\)-way interactions between all components \(C_m \triangleright 1 \leq m \leq k\)
6. for \(m = 1\) to \(k\) do
7. for \(j = 1\) to \(v_m\) do
8. Store the number of uncovered interactions of value \(val_{m,j}\) of component \(C_m\) in \(N_{\text{uncovered}}(val_{m,j})\)
9. end for
10. end for
11. for each component \(C_m\) do
12. Assign each value \(val_{m,j}\) an equal probability of selection \(P(val_{m,j})\)
13. end for
14. \(\triangleright \) Generate first test case
15. Create the first test case \(tc_{fi}\) by selecting a value for each component \(C_m\) randomly
16. Let \(val_{ms}\) is the selected value of component \(C_m\)
17. \(\triangleright \) Generate remaining \((N - 1)\) test cases
18. Initialize \(i := 2\)
19. for \(i = 2\) to \(N\) do
20. for \(m = 1\) to \(k\) do
21. Update interaction list \(L\) by eliminating the interactions covered by \(val_{ms}\) in test case \(tc_{fi-1}\)
22. Store the number of remaining interactions of \(val_{ms}\) in \(N'_{\text{uncovered}}(val_{ms})\)
23. Decrease probability of selection of value \(val_{ms}\) of \(C_m\) selected in test case \(tc_{fi-1}\) according to equation 3
24. Update probability of remaining values of \(C_m\) according to equation 5.
25. \end for
26. for each component \(C_m\) do
27. Generate a random number between 1 to 100
28. Create test case \(tc_{fi}\) by selecting a value of \(C_m\) based on the interval in which the random number falls
29. end for
30. end for
31. end for
32. Calculate fitness of each VSCA \(f\) in \(pop_1\) using equation 1
33. \(G \leftarrow G + 1\)
34. while solution not found and \(G \leq NOG\) do
35. \(\triangleright \) Perform Crossover
36. Initialize \(counter := 1\)
37. while \(counter \leq NOR\) do
38. Select two parent VSCA from population \(pop_{G-1}\) using Roulette Wheel Selection
39. Let \(parent_1\) is the parent VSCA having higher fitness among the two parents
40. Calculate the number of distinct pairs covered by each test case of \(parent_1\)
41. if \(NTC < \) number of test cases covering least number of distinct interactions then
42. Calculate the number of interactions covered by respective test cases of \(parent_2\)
43. Select NTC test cases of \(parent_2\) that cover maximum number of interactions not covered by \(parent_1\)
44. else if \(NTC > \) number of test cases covering least number of distinct interactions then
45. Select NTC test cases in \(parent_1\) when sorted in ascending order by the number of distinct interactions covered by them
else

    Select \(N\) \(TC\) test cases that covers least number of distinct interactions in \(parent_1\)

end if

Perform crossover between \(parent_1\) and \(parent_2\) by exchanging selected test cases to generate offsprings \(os_1, \, os_2\)

\[\text{▷ Perform Mutation}\]

Apply greedy mutation on \(os_1\) and \(os_2\) as discussed in Section 5.3

Replace weaker VSCA in \(pop_{G-1}\) by \(os_1, \, os_2\) to form new population \(pop_G\)

\[\text{counter} \leftarrow \text{counter} + 1\]

end while

Calculate fitness of each VSCA\(_f\) in the \(pop_G\) using equation 1

if solution found then

    break

else

    \[G \leftarrow G + 1\]

end if

if generations > \(NOG\) then

    return (solution not found)

else

    return VSCA\(_f\) \[\text{▷ VSCA with 100% fitness}\]

end if

end procedure